Supplement 4

Reliability

Learning Objectives

You should be able to:

LO 4s.1 Define *reliability*

LO 4s.2 Perform simple reliability computations

LO 4s.3 Explain the term *availability* and perform simple calculations

Reliability

- **Reliability**
  - The ability of a product, part, or system to perform its intended function under a prescribed set of conditions. Reliability can be expressed as a probability.

Quantifying Reliability

The probability that a system or a product will operate as planned is an important concept in system and product design. Thus, the corresponding reliability can be expressed as a probability:

- The probability that the product or system will function when activated
- The probability that the product or system will function for a given length of time

Reliability – When Activated

- Finding the probability under the assumption that the system consists of a number of independent components
  - Requires the use of probabilities for independent events
    - **Independent event**
      - Events whose occurrence or non-occurrence do not influence one another

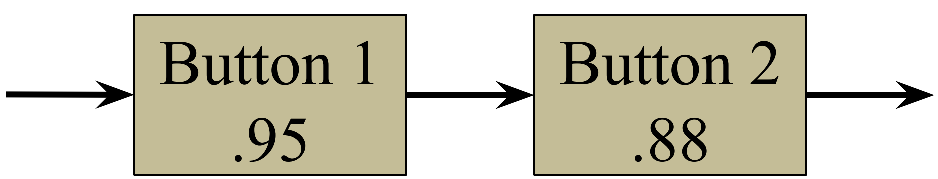
Reliability – When Activated (2 of 5)

- **Rule 1**
  - If two or more events are independent and *success* is defined as the probability that all of the events occur, then the probability of success is equal to the product of the probabilities of the events

Example – Rule 1

- A machine has two buttons. In order for the machine to function, both buttons must work. One button has a probability of working of .95, and the second button has a probability of working of .88.

$$P(\text{Machine Works}) = P(\text{Button 1 Works}) \times P(\text{Button 2 Works})$$
$$= .95 \times .88$$
$$= .836$$



LO 4s.2

Reliability – When Activated (3 of 5)

- Though individual system components may have high reliabilities, the system's reliability may be considerably lower because all components that are in series must function
- One way to enhance reliability is to utilize redundancy
  - **Redundancy**
    - The use of backup components to increase reliability

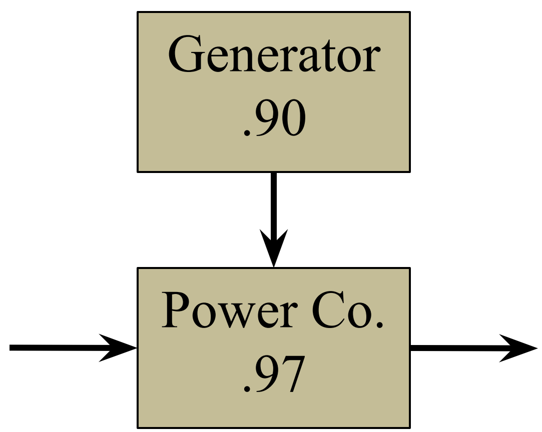
Reliability - When Activated (4 of 5)

- **Rule 2**
  - If two events are independent and *success* is defined as the probability that *at least one* of the events will occur, the probability of success is equal to the probability of either one plus 1.00 minus that probability multiplied by the other probability

Example – Rule 2

- A restaurant located in an area that has frequent power outages has a generator to run its refrigeration equipment in case of a power failure. The local power company has a reliability of .97, and the generator has a reliability of .90. The probability that the restaurant will have power is

$$
\begin{aligned}
P(\text{Power}) &= P(\text{Power Co.}) + (1 - P(\text{Power Co.})) \times P(\text{Generator}) \\
&= .97 + (1 - .97)(.90) \\
&= .997
\end{aligned}
$$



LO 4s.2

Reliability – When Activated (5 of 5)

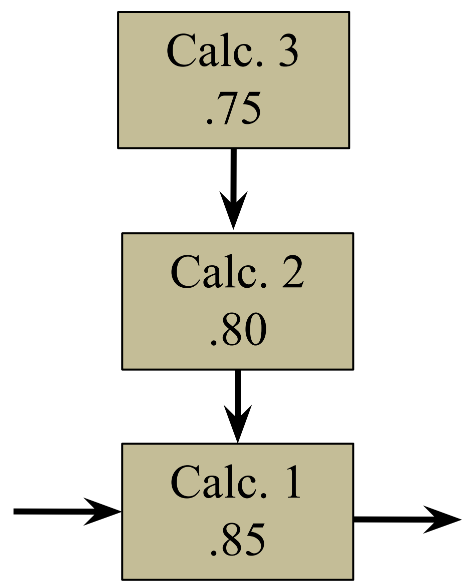
- **Rule 3**
  - If two or more events are involved and success is defined as the probability that at least one of them occurs, the probability of success is 1 – *P*(all fail)

LO 4s.2

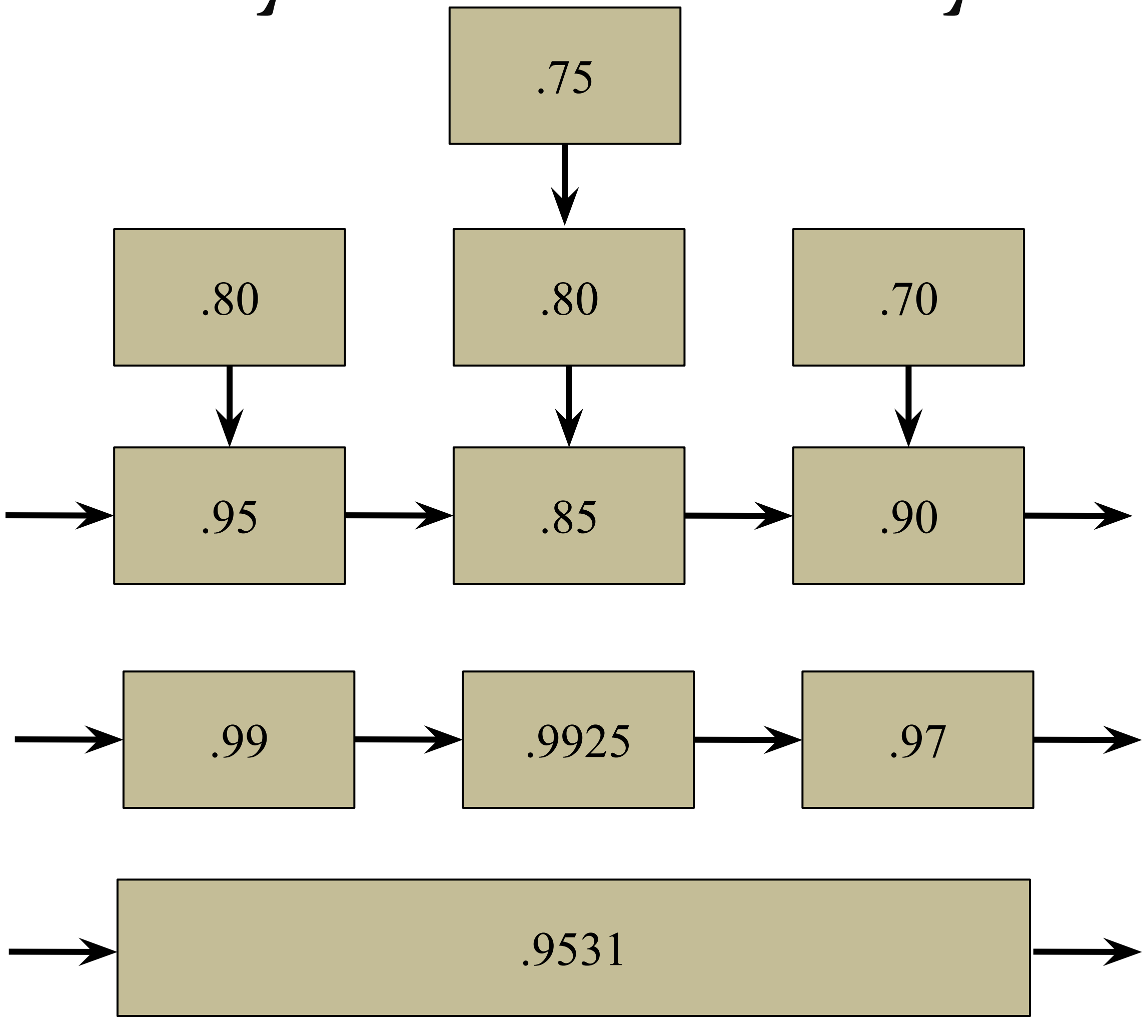
Example – Rule 3

- A student takes three calculators (with reliabilities of .85, .80, and .75) to her exam. Only one of them needs to function for her to be able to finish the exam. What is the probability that she will have a functioning calculator to use when taking her exam?

$$
\begin{aligned}
P(\text{any Calc.}) &= 1-[(1 - P(\text{Calc. }1)\times(1-P(\text{Calc. }2)\times(1-P(\text{Calc. }3)] \\
&= 1-[(1-.85)(1-.80)(1-.75)] \\
&= .9925
\end{aligned}
$$



What Is This System's Reliability?



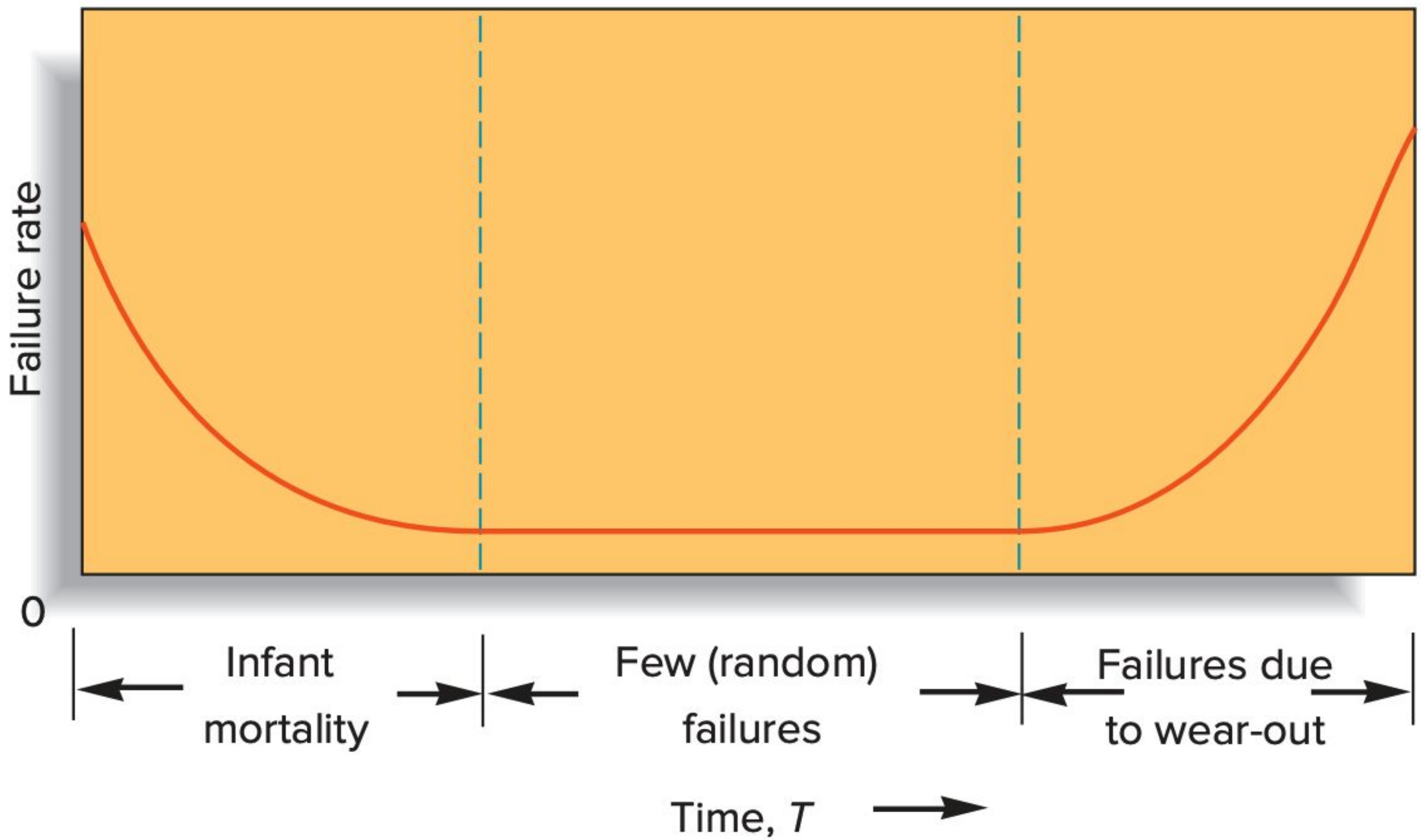
LO 4s.2

Reliability – Over Time

- In this case, reliabilities are determined relative to a specified length of time
- This is a common approach to viewing reliability when establishing warranty periods

LO 4s.2

The Bathtub Curve

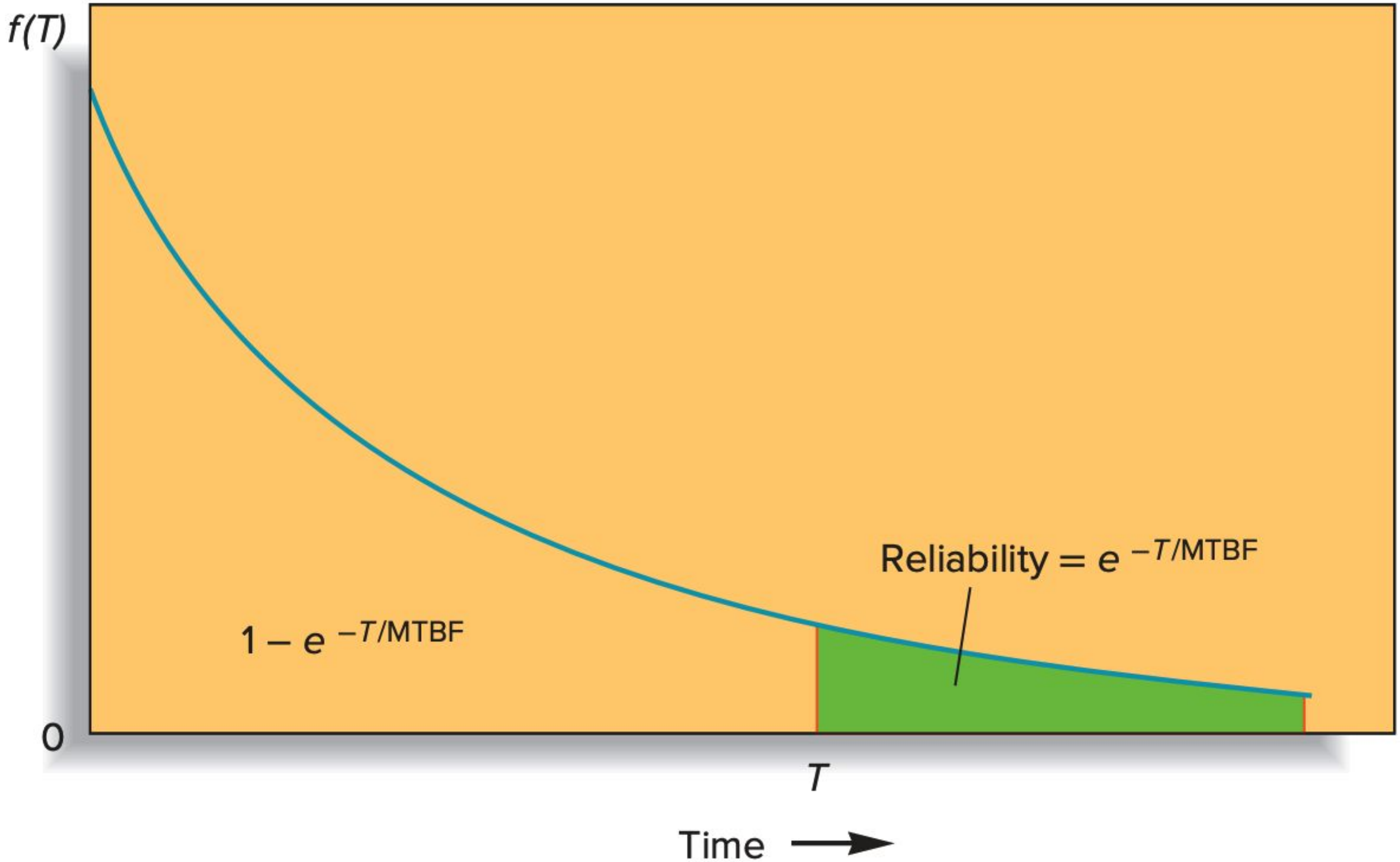


LO 4s.2

Distribution and Length of Phase

- To properly identify the distribution and length of each phase requires collecting and analyzing historical data
- The mean time between failures (MTBF) in the infant mortality phase can often be modeled using the negative exponential distribution

Exponential Distribution



LO 4s.2

Exponential Distribution - Formulae

$$P(\text{no failure before } T) = e^{-T/MTBF}$$

where

$$e = 2.7183...$$

$$T = \text{Length of service before failure}$$

$$\text{MTBF} = \text{Mean time between failures}$$

LO 4s.2

Example – Exponential Distribution

- A light bulb manufacturer has determined that its 150 watt bulbs have an exponentially distributed mean time between failures of 2,000 hours. What is the probability that one of these bulbs will fail before 2,000 hours have passed?

$$P(\text{failure before } 2{,}000) = 1 - e^{-2000/2000}$$

$e^{-2000/2000} = e^{-1}$

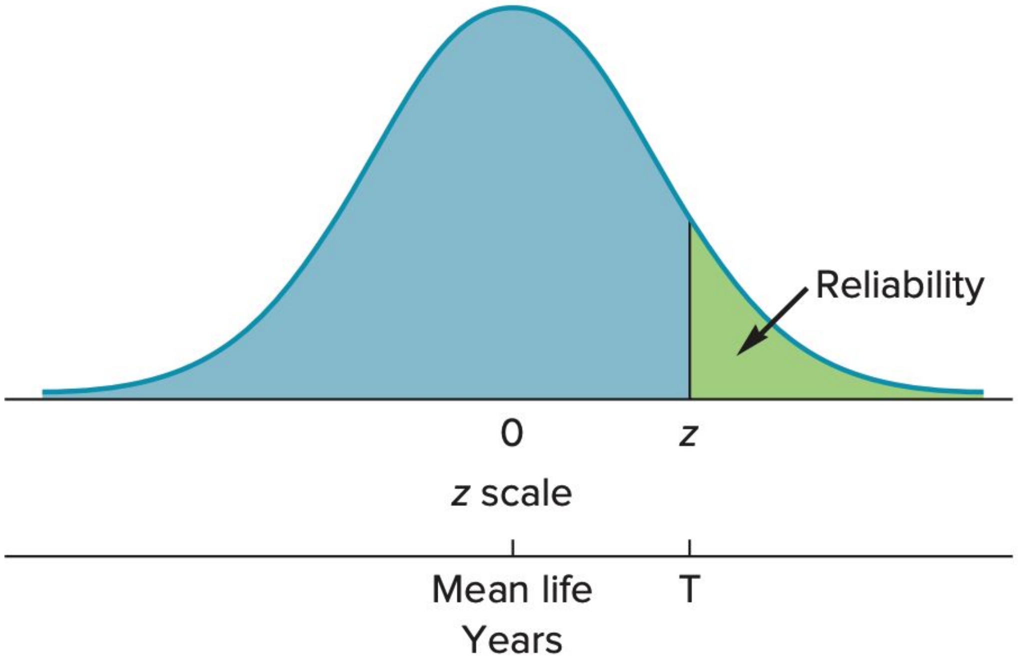
From Table 4S.1, $e^{-1} = .3679$

So, the probability one of these bulbs will fail before 2,000 hours is 1 – .3679 = .6321

LO 4s.2

Normal Distribution

- Sometimes, failures due to wear-out can be modeled using the normal distribution



$$z = \frac{T - \text{Mean wear-out time}}{\text{Standard deviation of wear-out time}}$$

LO 4s.2

Availability

- **Availability**
  - The fraction of time a piece of equipment is expected to be available for operation

$$\text{Availability} = \frac{\text{MTBF}}{\text{MTBF} + \text{MTR}}$$

where

$$\text{MTBF} = \text{Mean time between failures}$$

$$\text{MTR} = \text{Mean time to repair}$$

LO 4s.3

Example – Availability

- John Q. Student uses a laptop at school. His laptop operates 30 weeks on average between failures. It takes 1.5 weeks, on average, to put his laptop back into service. What is the laptop's availability?

$$\text{Availability} = \frac{\text{MTBF}}{\text{MTBF} + \text{MTR}}$$

$$= \frac{30}{30 + 1.5}$$

$$= .9524$$

LO 4s.3

Supplement 5

Decision Theory

Supplement 5: Learning Objectives

You should be able to:

LO 5s.1	Outline the steps in the decision process
LO 5s.2	Name some causes of poor decisions
LO 5s.3	Describe and use techniques that apply to decision making under uncertainty
LO 5s.4	Describe and use the expected-value approach
LO 5s.5	Construct a decision tree and use it to analyze a problem
LO 5s.6	Compute the expected value of perfect information
LO 5s.7	Conduct sensitivity analysis on a simple decision problem

Decision Theory

- A general approach to decision making that is suitable to a wide range of operations management decisions
  - Capacity planning
  - Product and service design
  - Equipment selection
  - Location planning

Characteristics of Suitable Problems

- Characteristics of decisions that are suitable for using decision theory
  - A set of possible future conditions that will have a bearing on the results of the decision
  - A list of alternatives from which to choose
  - A known payoff for each alternative under each possible future condition

Process for Using Decision Theory

1. Identify the possible future *states of nature*
2. Develop a list of possible *alternatives*
3. Estimate the *payoff* for each alternative for each possible future state of nature
4. If possible, estimate the *likelihood* of each possible future state of nature
5. Evaluate alternatives according to some *decision criterion* and select the best alternative

Payoff Table

- A table showing the expected payoffs for each alternative in every possible state of nature

	Possible Future Demand		
Alternatives	**Low**	**Moderate**	**High**
Small facility	$10	$10	$10
Medium facility	7	12	12
Large Facility	(4)	2	16

- A decision is being made concerning which size facility should be constructed
- The present value (in millions) for each alternative under each state of nature is expressed in the body of the above payoff table

Decision Process

- **Steps:**
  1. Identify the problem
  2. Specify objectives and criteria for a solution
  3. Develop suitable alternatives
  4. Analyze and compare alternatives
  5. Select the best alternative
  6. Implement the solution
  7. Monitor to see that the desired result is achieved

Causes of Poor Decisions

- Decisions occasionally turn out poorly due to unforeseeable circumstances; however, this is not the norm
- More frequently poor decisions are the result of a combination of
  - Mistakes in the decision process
  - Bounded rationality
  - Suboptimization

Mistakes in the Decision Process

- **Errors in the Decision Process**
  - Failure to recognize the importance of each step
  - Skipping a step
  - Failure to complete a step before jumping to the next step
  - Failure to admit mistakes
  - Inability to make a decision

Bounded Rationality & Suboptimization

- **Bounded rationality**
  - The limitations on decision making caused by costs, human abilities, time, technology, and availability of information
- **Suboptimization**
  - The results of different departments each attempting to reach a solution that is optimum for that department

Decision Environments

- **There are three general environment categories:**
  - **Certainty**
    - Environment in which relevant parameters have known values
  - **Risk**
    - Environment in which certain future events have probabilistic outcomes
  - **Uncertainty**
    - Environment in which it is impossible to assess the likelihood of various possible future events

Decision Making Under Certainty

- **Sometimes we know the exact outcome or environment. Under those situations, making a decision is easy. For example, if Investment 1 gives a return of 3.4% and Investment 2 gives a return of 5.6%, then we know what to do.**
- **Uncertainty comes when there is a risk involved, for instance, Investment 1 could be the return on a CD which is guaranteed, but Investment 2 could be a mutual fund whose returned can't be guaranteed.**

Decision Making Under Uncertainty

- **Decisions are sometimes made under complete uncertainty: No information is available on how likely the various states of nature are.**
- **Decision criteria:**
  - **Maximin**
    - Choose the alternative with the best of the worst possible payoffs
  - **Maximax**
    - Choose the alternative with the best possible payoff
  - **Laplace**
    - Choose the alternative with the best average payoff
  - **Minimax regret**
    - Choose the alternative that has the least of the worst regrets

LO 5s.3

Example – Maximin Criterion

	Possible Future Demand		
Alternatives	Low	Moderate	High
Small Facility	$10	$10	$10
Medium Facility	7	12	12
Large Facility	(4)	2	16

- The worst payoff for each alternative is
  - Small facility: $10 million
  - Medium facility $7 million
  - Large facility -$4 million
- **Choose to construct a small facility**

LO 5s.3

Example – Maximax Criterion

	Possible Future Demand		
Alternatives	Low	Moderate	High
Small Facility	$10	$10	$10
Medium Facility	7	12	12
Large Facility	(4)	2	16

- The best payoff for each alternative is
  - Small facility: $10 million
  - Medium facility $12 million
  - Large facility $16 million
- **Choose to construct a large facility**

LO 5s.3

Example – Laplace Criterion

	Possible Future Demand		
Alternatives	Low	Moderate	High
Small Facility	$10	$10	$10
Medium Facility	7	12	12
Large Facility	(4)	2	16

- The average payoff for each alternative is
  - Small facility: (10 + 10 + 10) ÷ 3 = $10 million
  - Medium facility (7 + 12 + 12) ÷ 3 = $10.33 million
  - Large facility (−4 + 2 + 16) ÷ 3 = $4.67 million
- **Choose to construct a medium facility**

LO 5s.3

Example – Minimax Regret

	Possible Future Demand		
Alternatives	Low	Moderate	High
Small Facility	$10	$10	$10
Medium Facility	7	12	12
Large Facility	(4)	2	16

- Construct a **regret** (or **opportunity loss**) table
    - The difference between a given payoff and the best payoff for a state of nature

	Regrets		
Alternatives	Low	Moderate	High
Small Facility	$0	$2	$6
Medium Facility	3	0	4
Large Facility	14	10	0

LO 5s.3

Example – Minimax Regret (cont.)

	Regrets		
Alternatives	Low	Moderate	High
Small Facility	$0	$2	$6
Medium Facility	3	0	4
Large Facility	14	10	0

- Identify the worst regret for each alternative
  - Small facility $6 million
  - Medium facility $4 million
  - Large facility $14 million
- Select the alternative with the minimum of the maximum regrets
  - **Build a medium facility**

Decision Making Under Risk

- **Decisions made under the condition that the probability of occurrence for each state of nature can be estimated**
- **A widely applied criterion is expected monetary value (EMV)**
  - EMV
    - Determine the expected payoff of each alternative, and choose the alternative that has the best expected payoff
  - This approach is most appropriate when the decision maker is neither risk averse nor risk seeking

Example – EMV

	Possible Future Demand		
Alternatives	Low (.30)	Moderate (.50)	High (.20)
Small Facility	$10	$10	$10
Medium Facility	7	12	12
Large Facility	(4)	2	16

$EMV_{small} = .30(10) + .50(10) + .20(10) = 10$

$EMV_{medium} = .30(7) + .50(12) + .20(12) = 10.5$

$EMV_{large} = .30(-4) + .50(2) + .20(16) = \$3$

Build a medium facility

Decision Tree

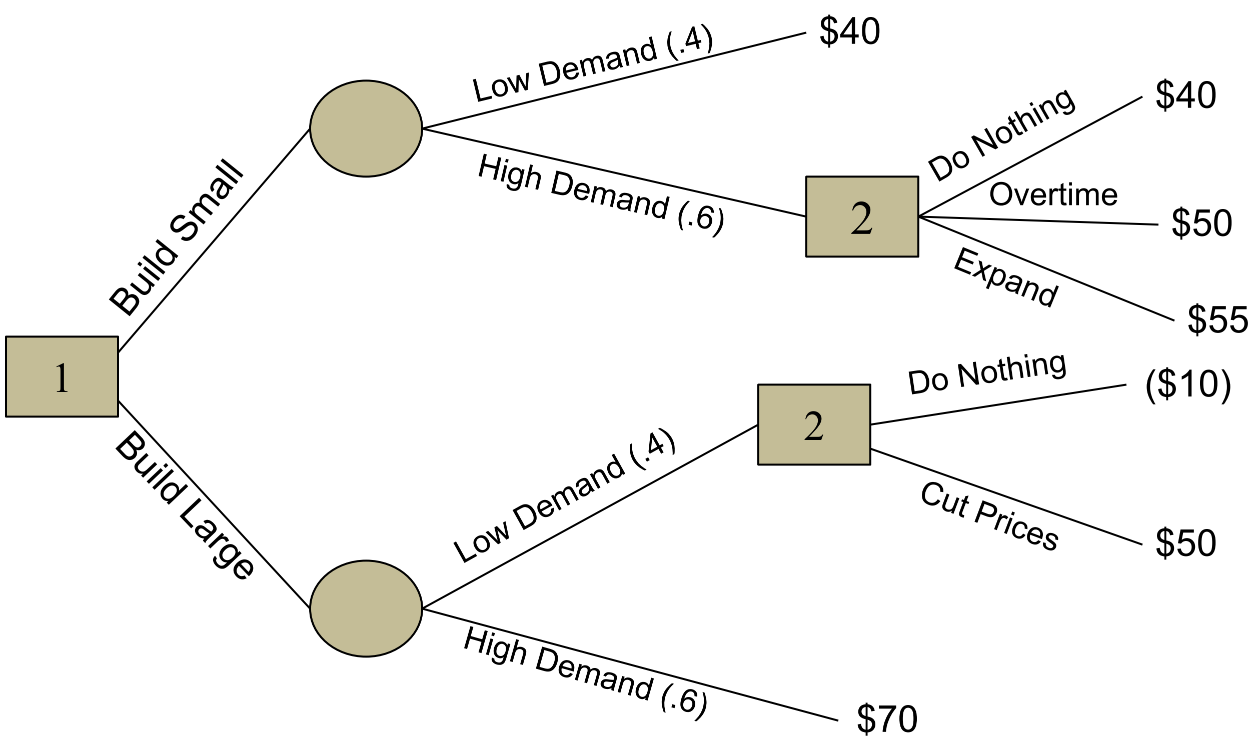
- **Decision tree**
  - A schematic representation of the available alternatives and their possible consequences
  - Useful for analyzing sequential decisions

Decision Tree (cont.)

- **Composed of**
  - Nodes
    - Decisions – represented by square nodes
    - Chance events – represented by circular nodes
  - Branches
    - Alternatives – branches leaving a square node
    - Chance events – branches leaving a circular node
- **Analyze from right to left**
  - For each decision, choose the alternative that will yield the greatest return
  - If chance events follow a decision, choose the alternative that has the highest expected monetary value (or lowest expected cost)

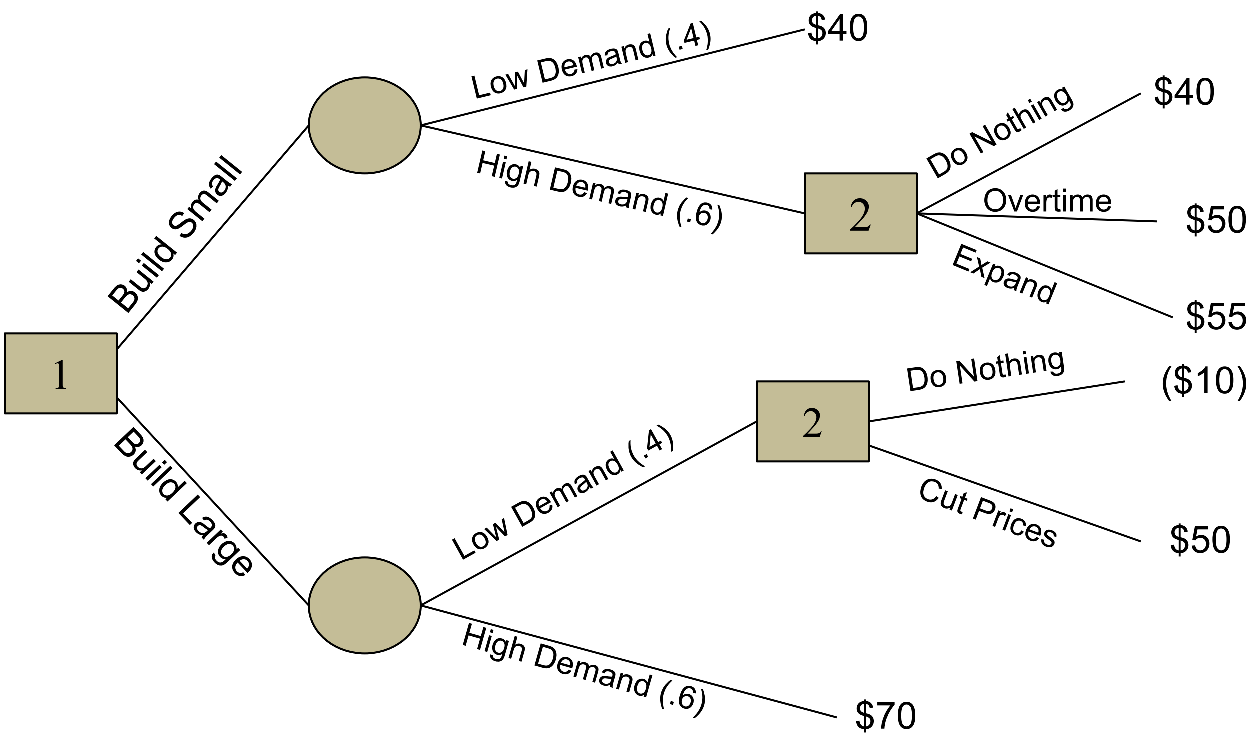
Example – Decision Tree

- A manager must decide on the size of a video arcade to construct. The manager has narrowed the choices to two: large or small. Information has been collected on payoffs, and a decision tree has been constructed. Analyze the decision tree and determine which initial alternative (build small or build large) should be chosen in order to maximize expected monetary value.



LO 5s.5

Example – Decision Tree (cont.)



$$EV_{Small} = .4(40) + .6(55) = \$49$$
$$EV_{Large} = .4(50) + .6(70) = \$62$$

Build the large facility

LO 5s.5

Expected Value of Perfect Information

- **Expected value of perfect information (EVPI)**
  - The difference between the expected payoff with perfect information and the expected payoff under risk
  - Two methods for calculating EVPI
    - EVPI = expected payoff under certainty – expected payoff under risk
    - EVPI = minimum expected regret

Example – EVPI

	Possible Future Demand		
Alternatives	Low (.30)	Moderate (.50)	High (.20)
Small Facility	$10	$10	$10
Medium Facility	7	12	12
Large Facility	(4)	2	16

$EV_{\text{with perfect information}} = .30(10) + .50(12) + .20(16) = \$12.2$

EMV = $10.5

$EVPI = EV_{\text{with perfect information}} - EMV$

$= \$12.2 - 10.5$

$= \$1.7$

You would be willing to spend up to $1.7 million to obtain perfect information

LO 5s.6

Example – EVPI (cont.)

	Regrets		
Alternatives	Low (.30)	Moderate (.50)	High (.20)
Small Facility	$0	$2	$6
Medium Facility	3	0	4
Large Facility	14	10	0

- Expected Opportunity Loss
  - $EOL_{Small} = .30(0) + .50(2) + .20(6) = \$2.2$
  - $EOL_{Medium} = .30(3) + .50(0) + .20(4) = \$1.7$
  - $EOL_{Large} = .30(14) + .50(10) + .20(0) = \$9.2$

- The minimum EOL is associated with the building the medium size facility. **This is equal to the EVPI, $1.7 million.**

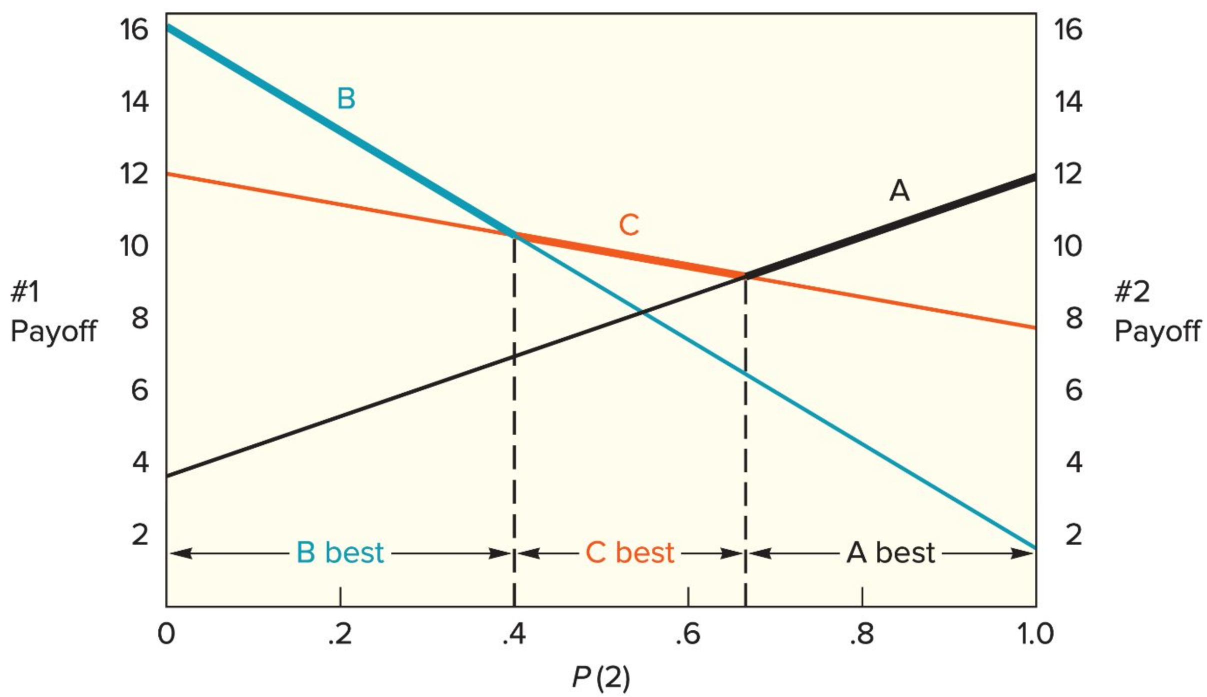
LO 5s.6

Sensitivity Analysis

- **Sensitivity analysis**
  - Determining the range of probability for which an alternative has the best expected payoff
  - The approach illustrated is useful when there are two states of nature
    - It involves constructing a graph and then using algebra to determine a range of probabilities over which a given solution is best

Sensitivity Analysis (cont.)

	State of Nature			
Alternative	#1	#2	Slope	Equation
A	4	12	12 − 4 = +8	4 + 8P(2)
B	16	2	2 − 16 = −14	16 − 14P(2)
C	12	8	8 − 12 = −4	12 − 4P(2)

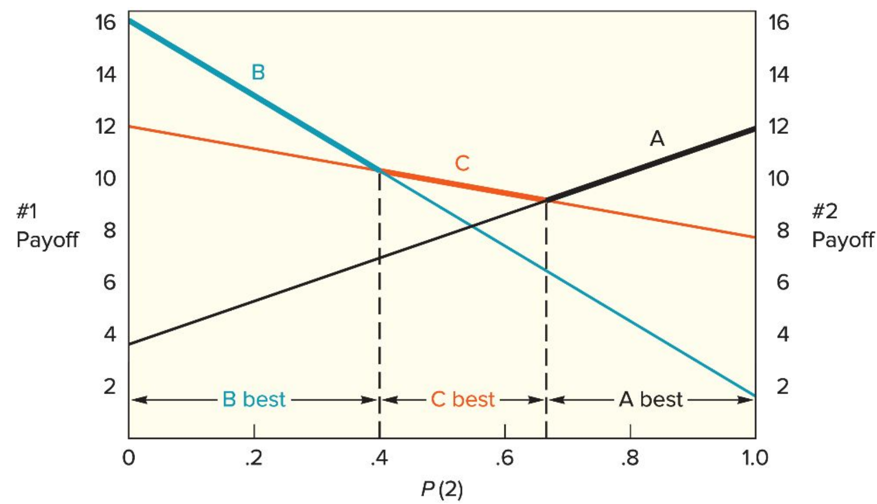


LO 5s.7

Sensitivity Analysis (Solution steps)

$16-14P(2)=12-4P(2)$
Rearranging terms yields
$4=10P(2)$
Solving yields $P(2) = .40$.
Thus, alternative B is best from $P(2) = 0$ up to $P(2) = .40$. B and C are equivalent at $P(2) = .40$.

Similar analysis can be used for alternative A and C
$4+8P(2)=12-4P(2)$
Solving yields $P(2) = .67$.
Thus, alternative C is best from $P(2) > .40$ up to $P(2) = .67$, where A and C are equivalent. For values of $P(2)$ greater than .67 up to $P(2) = 1.0$, A is best.



LO 5s.7